DETERMINATION OF THE MAIN FLOW PARAMETERS IN A SWIRL SPRAYER BY MEANS OF CONSERVATION LAWS

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The principle of maximum flow rate (PMR) proposed by Abramovich [1, 2] is now widely used to calculate the main flow parameters in swirl sprayers. Here we show that the basic flow parameters in a swirl sprayer of a special type (a sprayer with a Borda mouthpiece) can be calculated exactly by means of conservation laws within the framework of the model of an ideal incompressible fluid. Comparison of the exact results with the results obtained on the basis of the PMR shows that they differ substantially.

Figure 1 shows a sketch of the swirl sprayer and the pattern of flow within it. Fluid flowing through the sprayer is directed to the swirl chamber along cylindrical tangential channels of the radius r_0. The axes of these channels are displaced relative to the sprayer axis, which coincides with the z axis of the cylindrical coordinate system. The channel axes are displaced by the amount R_0 and are located in a plane perpendicular to the z axis. The fluid acquires an axial angular momentum, and intensive rotation begins to occur. A hollow vortex is formed on the sprayer axis. The vortex has the radius R_1 on the rear wall of the swirl chamber and the radius R_2 on the straight section of the nozzle where the flow is equalized (the axial velocity component is independent of z). The pressure p at the boundary of the cavity takes a constant value which can be assumed to be equal to zero. At the outlet of the nozzle (mouthpiece) of radius R, the liquid is dispersed and forms a spray with the angle α. In reality, a film of liquid is broken up into drops, with the size of the drops depending to a significant extent on the thickness of the liquid layer δ on the straight section of the nozzle. The value of δ is determined by the nozzle fullness factor $\delta = R(1 - \sqrt{1 - \varphi})$, $\varphi = 1 - (R_2/R)^2$.

The main parameters of the flow in the swirl sprayer (the most important characteristics of the sprayer from the viewpoint of its practical use) are as follows: the flow rate for the specified pressure gradient at the inlet and outlet, defined by the discharge coefficient μ; the nozzle fullness factor φ; the spray angle α.

In the swirl sprayers in actual use, the flow and the above-mentioned quantities depend to a considerable extent on the viscosity of the liquid (the possibility of the onset of turbulence must also be taken into account).

Nevertheless, it is of definite interest to study flow in swirl sprayers within the framework of the theory of an ideal incompressible fluid. Such an approach provides a basis for understanding the most characteristic features of the phenomenon in question. It makes it possible to find the main flow parameters in a first approximation and is obviously a

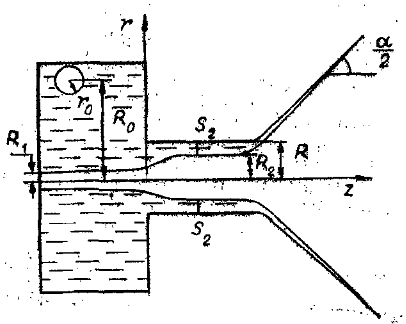

Fig. 1

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necessary step toward the development of reliable engineering methods of designing actual swirl sprayers. Comparison of the results of the theory and experiment without consideration of viscosity and turbulence may have some usefulness for several existing sprayers and some designs of special sprayers (which have yet to be built).

Within the framework of the model of an ideal incompressible fluid, steady-state axisymmetric flow with swirling (as we can consider the flow in a swirl sprayer to be, in a first approximation) is described by the following equation for the stream function written in a cylindrical coordinate system using standard notation [3]:

$$\frac{\partial^2\psi}{\partial z^2}+\frac{\partial^2\psi}{\partial r^2}-\frac{1}{r}\frac{\partial\psi}{\partial r}=r^2\frac{dH}{d\psi}-\frac{\Gamma}{4\pi^2}\frac{d\Gamma}{d\psi}, \tag{1}$$

where $H(\psi)$ and $\Gamma(\psi)$ are arbitrary functions of ψ; $H(\psi)$ is the right side of the Bernoulli integral, it being the consequence of the energy conservation law

$$\frac{1}{2}(v_z^2+v_\varphi^2+v_r^2)+\frac{p}{\rho}=H(\psi); \tag{2}$$

$\Gamma(\psi)$ is the circulation about a fluid contour in the form of a circle having its center on the z axis and located in a plane perpendicular to this axis. Thus,

$$v_\varphi=\Gamma(\psi)/2\pi r. \tag{3}$$

It is evident that Eq. (3) expresses the law of conservation of angular momentum. To determine the function $\psi(r, z)$ from Eq. (1), it is necessary to specify the form of the functions $H(\psi)$ and $\Gamma(\psi)$. This can be done by assuming that they are known at the inlet. It is also necessary to assign the corresponding boundary conditions: the condition of impermeability on the solid walls; constancy of pressure; the kinematic condition on the free surface of the vortex.

However, it is known that stagnant regions with closed streamlines and reverse flows may develop in the flow being examined, and assigning $H(\psi)$ and $\Gamma(\psi)$ in these cases requires special investigation and the use of additional hypotheses. If such hypotheses have been worked out and $H(\psi)$ and $\Gamma(\psi)$ have been assigned in these regions, then additional hypotheses of the PMR type are not needed to determine the flow. Due to the nonlinearity of Eq. (1), the problem may also not have a unique solution under these conditions. In this case, solutions to be compared with experimental results are selected on the basis of additional analysis (study of stability, changing of flow regimes, analysis of the problem with initial conditions, etc.).

However, except for certain very simple cases, such an approach to studying a specific problem is very complicated and, in the case of flow in a swirl sprayer, is not used. Instead, the flow parameters in swirl sprayers are commonly calculated by an approach based (within the framework of the ideal incompressible fluid model) on the use of conservation laws and the PMR (or other similar principles).

The theory in [1, 2] (Taylor [4] and many other investigators later arrived at similar conclusions) is based on the following premises. The quantities $H(\psi)$ and $\Gamma(\psi)$ are independent of ψ and are constant:

$$H(\psi)=p_0/\rho,\ \Gamma(\psi)=\Gamma \tag{4}$$

[p_0 is the pressure (total head) at the inlet of the sprayer]. As is readily apparent, these propositions correspond to the presumption of potential flow.

It follows from this that the axial component of velocity v_z in the nozzle on the section where the flow is equalized is independent of r, i.e., is constant over the cross section of the flow S_2. Let $v_z = w$ in this section. Then we find from the mass conservation law that

$$Q=\pi R^2\varphi w. \tag{5}$$

With allowance for the law of conservation of angular momentum (3), the law of energy conservation or the equivalent Bernoulli integral (2) gives the following for the free surface in the same section:

$$\frac{1}{2}w^2+\frac{\Gamma^2}{8\pi^2 R^2}=\frac{p_0}{\rho}. \tag{6}$$

The design of the swirl sprayer leads to the following relationship between Γ, Q, and R:

$$\Gamma = 2AQ/R, \tag{7}$$

where the dimensionless quantity A (the flow swirl parameter) is the main geometric characteristic of a swirl sprayer and is determined by the sprayer's geometric parameters [1]: $A = R_0R/(nr_0^2)$ (n is the number of inlet channels).

Taking (1) and (2) into account, we use (5) and (6) to obtain:

$$Q = \mu\pi R^2 \sqrt{\frac{2p_0}{\rho}}; \tag{8}$$

$$\mu = \frac{\varphi\sqrt{1-\varphi}}{\sqrt{1-\varphi+A^2\varphi^2}} \tag{9}$$

(μ is the discharge coefficient).

It is evident from this that use of the three conservation laws (mass, energy, and angular momentum) does not allow us to find the main parameters of the flow. The quantity φ remains unknown. Use of the law of conservation of momentum for the sprayer sketched in Fig. 1 does not ensure closure, since the pressure distribution on the walls of the sprayer is unknown.

The PMR hypothesis consists of the fact that the hollow vortex formed in the nozzle of a swirl sprayer has a radius such that μ takes the maximum value for the given head p_0. These vortex dimensions correspond to a stable flow regime.

The hypothesis leads to relations which make it possible to find μ and φ as a function of the swirl parameter A:

$$\mu = \frac{\varphi^{3/2}}{\sqrt{2-\varphi}}, \quad A = \frac{\sqrt{2}\,(1-\varphi)}{\varphi^{3/2}}. \tag{10}$$

The spray angle α was determined in [1] as the mean value of the ratio of the azimuthal and axial velocity components across the nozzle

$$\mathrm{tg}\frac{\alpha}{2} = \frac{\langle v_\varphi\rangle}{w}, \quad \langle v_\varphi\rangle = \frac{\Gamma}{2\pi R}\frac{2R}{R+R_2}, \tag{11}$$

from which

$$tg\frac{\alpha}{2} = \frac{2A\varphi}{1+\sqrt{1-\varphi}}. \tag{12}$$

Figure 2 (dashed lines) shows the dependences of μ, φ, and α on A constructed by means of Eqs. (10) and (12).

Using the analogy of the flow of a heavy liquid through a spillway, the author of [5] interpreted the PMR as the condition of equality of the velocity in the nozzle of a swirl sprayer to the maximum velocity of the centrifugal waves (long low-amplitude waves propagating over the surface of the hollow potential vortex in the cylindrical channel). The velocity of the centrifugal waves on the free surface of the hollow vortex in the nozzle is found from the formula

$$w_* = \frac{\Gamma}{2\pi R}\sqrt{\frac{R^2-R_2^2}{2}}. \tag{13}$$

Taking (7) into account, we find from (5) and (13) that

$$s = \frac{w}{w_*} = \frac{\sqrt{2}}{A}\frac{1-\varphi}{\varphi^{3/2}}. \tag{14}$$

Thus, it follows from the PMR that s = 1. The equivalence of the PMR and the requirement that the condition s = 1 be satisfied is regarded as validation of the PMR.

The arguments made in support of the PMR are unconvincing. They have been the subject of repeated criticism [6, 7], but the criticisms themselves are somewhat questionable.

It would be useful and instructive to look at an example (albeit one that is to some extent artificial) of flow in which the main parameters are determined exactly. We then compare these results with the solution based on the PMR. Such an example is presented below.

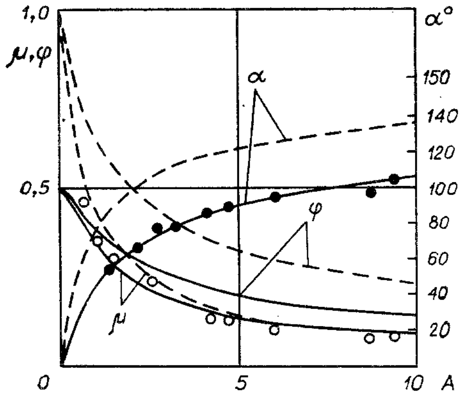

Fig. 2

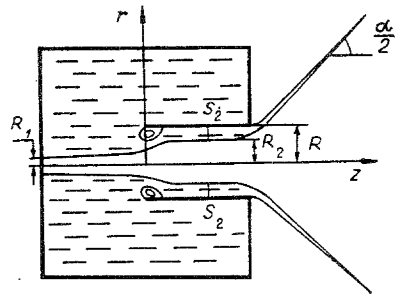

Fig. 3

We take a swirl sprayer with a Borda mouthpieze (nozzle) (Fig. 3). As is known, the specific form of a Borda mouthpiece in the case of the discharge of a jet without swirling allows effective use of the law of momentum conservation and makes it possible to determine the cross-sectional area of the outgoing jet.

It is not hard to see that for swirled flow in a swirl sprayer with a Borda mouthpiece, the momentum conservation law makes it possible to obtain a closed system of equations to determine the main parameters of the flow without the use of additional hypotheses.

In the limiting case, when the end walls are moved away and the radius of the chamber is increased to infinity, the results obtained are exact.

It is assumed that the flow is a potential flow except for the region with closed streamlines which forms on the interior of the nozzle edge, where the flow structure does not affect the subsequent discussion.

We apply the momentum conservation law to fluid bounded by the free surface of a hollow vortex, the walls of the nozzle, and the walls of the swirl chamber. The vortex has the area S_2 and is normal to the flow axis. It is located in the section where the flow is equalized (at infinity). For the z component of the momentum flux we obtain

$$2\pi \int_{R_1}^{R} p_1(r)\, r dr - 2\pi \int_{R_2}^{R} p_2(r)\, r dr - \rho\pi R^2 \varphi w^2 = 0. \tag{15}$$

The remaining sections of the surface of integration do not contribute to this component or are self-balancing. In the limit, we have

$$p_1(r) = p_0 - \rho\Gamma^2/(8\pi^2 r^2); \tag{16}$$

$$p_2(r) = p_0 - \rho\Gamma^2/(8\pi^2 r^2) - (1/2)\rho w^2. \tag{17}$$

Here, $p_1(r)$ is the pressure on the front and rear walls of the swirl chamber; $p_2(r)$ is the pressure in the section S_2. The value of R_1 is determined from the condition whereby $p_1(R_1)$ vanishes due to the triviality of pressure on the free surface. This gives

$$R_1 = \Gamma/2\pi\sqrt{2p_0/\rho}. \tag{18}$$

Integrating in (15) with allowance for (16) and (17), we find

$$\frac{1}{2}\left[\frac{\varphi_1}{\mu^2} + A^2 \ln(1-\varphi_1)\right] = \frac{1}{\varphi} + \frac{1}{2}A^2\left[\frac{\varphi}{1-\varphi} + \ln(1-\varphi)\right], \tag{19}$$

where $\varphi_1 = 1 - (R_1/R)^2$, while from (18) we have

$$1 - \varphi_1 = A^2\mu^2. \tag{20}$$

Thus, using the laws of conservation of mass, momentum, angular momentum, and energy and boundary condition (18), we obtained closed system (9), (19), (20) to determine μ, φ, and φ_1.

The solution of this system is conveniently represented in the form

$$A^2=\frac{q(2+\lambda)}{(1-q)(1+\lambda)^2},\quad \lambda=\frac{q\ln q}{1-q},\quad \mu=\frac{1+\lambda}{2+\lambda}\sqrt{1-q},$$

$$\varphi=\frac{1+\lambda}{2+\lambda},\quad \varphi_1=1-\frac{q}{2+\lambda}. \tag{21}$$

Here, q changes within the range $0 < q < 1$. The relations $\mu(A)$ and $\varphi(A)$ calculated from these formulas are shown in Fig. 2 (solid curves). With a change in A from $A = 0$ to $A \to \infty$, μ and φ decrease monotonically from 1/2 to 0. The spray angle α can also be found by means of the momentum conservation law. We find that $v_\varphi \to 0$ in the spray at large distances from the edge of the nozzle, i.e., at large values of r. Thus, the axial component of the momentum flux I in the jet is given by the equality

$$I=\rho Q\sqrt{\frac{2p_0}{\rho}}\cos\frac{\alpha}{2}. \tag{22}$$

On the other hand, the same component of flux over the surface S_2 in the nozzle is

$$I=\rho\pi R^2\varphi w^2+\frac{\rho\Gamma^2}{8\pi}\left[\frac{\varphi}{1-\varphi}+\ln(1-\varphi)\right]. \tag{23}$$

Equating (22) and (23), we find

$$\cos\frac{\alpha}{2}=\frac{\sqrt{1-\varphi}}{\sqrt{1-\varphi+A^2\varphi^2}}\left\{1+\frac{1}{2}A^2\left[\frac{\varphi^2}{1-\varphi}+\varphi\ln(1-\varphi)\right]\right\}.$$

The relation $\alpha(A)$ is shown in Fig. 2 (solid curve).

Thus, for a swirl sprayer with a Borda mouthpiece, the conservation laws can be used to determine the basic flow parameters unambiguously. Meanwhile, for an ideal fluid in the limiting case, these results are exact and differ significantly from the results obtained on the basis of the PMR. It can be seen from Fig. 2 that the results for μ coincide at large A. With large degrees of swirling (large A), the pressure distribution on the walls of the swirl chambers in swirl sprayers with conventional nozzles will approach distribution (16). This makes it possible to use the momentum conservation law in this case (with an accuracy which increases as A increases) and to expect that the main flow parameters will be close to the values calculated for a sprayer with a Borda mouthpiece. Of course, evaluation of the accuracy of the approximation obtained by such an approach will require a special investigation, but a comparison with experimental results is of some use, particularly for large A.

Figure 2 shows experimental data [7] for a sprayer with $R = 0.35$ cm, a nozzle length of 0.5 cm, and $p_0 = 3\cdot 10^6$ Pa (the light circles correspond to μ, while the dark circles correspond to α). It is evident that the experimental points for a sprayer with such parameters agree with the results calculated in the present study. However, there is a considerable difference for sprayers with smaller R at lower values of p_0. This difference can evidently be attributed to the effect of viscosity.

The value of s determined from Eq. (14), equal to unity by the PMR, changes from $s \to \infty$ to $s = 2$ with a change in A from $A = 0$ to $A \to \infty$, i.e., the flow is supercritical. This produces the above-noted difference for φ, despite the closeness of the discharge coefficients at large A.

Thus, the principal premise of the PMR (that the flow in the nozzle must be exactly critical) is not consistent with the exact solution. This casts a doubt on the reliability of the results obtained using the PMR.

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DYNAMICS OF A UNIFORM TURBULENT LAYER IN A STRATIFIED FLUID

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At large Reynolds numbers, a flow of an incompressible density-stratified fluid separates into alternating layers of turbulent and laminar flow [1]. Turbulent flow develops under the influence of shear instability or as a result of boundary conditions. One problem encountered in the description of such flows is representing the process of entrainment of surrounding fluid into a turbulent layer in parametric form [2, 3]. The most important factor here is the effect of stratification on the rate of entrainment. Various mechanisms of instability development, leading to mixing [2], become predominant. The specific mechanism that prevails depends on the relation between buoyancy and inertia. When mixing occurs, the rate of entrainment may change by several orders of magnitude. Since the flow region in which a given type of instability will develop is unknown beforehand, it is interesting to attempt to construct a model of stratified flow that will uniquely describe the entrainment process.

One possible approach to the solution of this problem is demonstrated below by using the example of the evolution of a turbulent layer in a quiescent fluid of another density. This class of flows includes submerged jets, gravitational flows, and the movement of the uniform upper layer of an ocean to a lower depth by wind [1]. The model that is constructed should reflect such experimentally observed flow properties as the potential for controlling the entrainment process by altering the conditions downflow, the sharp reduction in entrainment velocity with the transition from supercritical to subcritical flow, and the phenomenon of the excitation of short internal waves at the boundary of the turbulent layer in flows with a velocity shift [2].

Here we examine these phenomena on the basis of equations of motion of the layer which constitute a variant of the equations of "shallow water." Allowance is made for mixing. The equations of motion were derived from conservation laws in a manner similar to [4]. The rate of entrainment of fluid into the turbulent layer is assumed to be proportional to the velocity of "large eddies" that are commensurate with the thickness of the layer [2, 5]. Analysis of traveling waves in the system in question shows that solutions of the solitary-wave or jump-wave types describe the naturally-observed generation of short-period internal waves at the crests of longer (tidal) waves [6, 7].

Equations of "Shallow Water." In the Boussinesq approximation, the equations of a thin horizontal layer of fluid of thickness h and density ρ moving at velocity u in a quiescent fluid of density ρ_r have the form

$$\begin{gathered} h_t + (hu)_x = \sigma q,\ (bh)_t + (bhu)_x = 0, \\ (hu)_t + (hu^2 + 0.5bh^2)_x = 0, \\ (h(u^2 + e + bh))_t + (hu(u^2 + e + 2bh))_x = 0. \end{gathered} \tag{1}$$

Here, t is time; x, horizontal coordinate; $b = (\rho - \rho_r)g/\rho_r$, buoyancy; g, the vertical component of acceleration due to gravity; e is the energy associated with pulsative motion. The rate of entrainment of the quiescent fluid into the uniform layer is assumed to be proportional to the velocity of "large eddies" q characterizing pulsative motions in the layer. The eddies are comparable in size to the main flow [5]. System (1) will be closed if we put

$$e = q^2. \tag{2}$$

The constant σ determines the ratio of the scales of fluctuational and average motions in

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